Big rationals

--> bigQ

Introduction

Big integers is a package developed by Overlord, which can do algebra operations with arbitrary precission over integers. A big integer number is a string with only numbers and only one or none minus sign and dot.

$m := \text{"6790837465238495847362950498374583249584723894"}$

$n := \text{"92948372764503927827304838327"}$

which may or may not have representation in SMath

$a := \mathrm{str2num}(m) = 6.790837465238500 \cdot 10^{45}$

$b := \mathrm{str2num}(n) = 9.294837276450390 \cdot 10^{28}$

Big integer algebra:

$\mathrm{bigMod}(m, n) = \text{"49435269666259970706032146345"}$

$\mathrm{bigDiv}(m, n) = \text{"73060315778135387"}$

$\mathrm{bigSub}(m, n) = \text{"6790837465238495754414577733870655422279885567"}$

bigQ are a sorted couple of big numbers

$r := \begin{bmatrix} \text{"3"} \\ \text{"4"} \end{bmatrix} \qquad s := \begin{bmatrix} \text{"12"} \\ \text{"5"} \end{bmatrix} \qquad a := \frac{3}{4} \qquad b := \frac{12}{5}$

with which you can do algebra and usual calculus, but now with an arbitrary precission, setting the constant 'Digits $\boxed{\mathrm{Digits} := 16}$

$R := \mathit{bigQSub}(r, s) = \begin{bmatrix} \text{"-33"} \\ \text{"20"} \end{bmatrix}$ $\qquad \mathit{bigQ2D}(R) = \text{"-1.650000000000000"}$

$a - b = -1.650000000000000$

$R := \mathit{bigQMul}(r, s) = \begin{bmatrix} \text{"9"} \\ \text{"5"} \end{bmatrix}$ $\qquad \mathit{bigQ2D}(R) = \text{"1.800000000000000"}$

$a \cdot b = 1.800000000000000$

$R := \mathit{bigQDiv}(r, s) = \begin{bmatrix} \text{"5"} \\ \text{"16"} \end{bmatrix}$ $\qquad \mathit{bigQ2D}(R) = \text{"0.312500000000000"}$

$\frac{a}{b} = 0.312500000000000$

$R := \mathit{bigQPow}(r, s) = \begin{bmatrix} \text{"252427356530549990486293902673644836600884139182398721387140976801386529\ldots} \\ \text{"503488304908147320986882340427719104791780456954472170744805181783583900\ldots} \end{bmatrix}$

$\mathit{bigQ2D}(R) = \text{"0.501356941302938"} \qquad a^{b} = 0.501356941302939$

$\mathit{bigQ2D}(\mathit{bigQSin}(r)) = \text{"0.681638760023334"} \qquad \sin(a) = 0.681638760023334$

$\mathit{bigQ2D}(\mathit{bigQLog}(r)) = \text{"-0.287682072451780"} \qquad \ln(a) = -0.287682072451781$

$\mathit{bigQ2D}(\mathit{bigQExp}(r)) = \text{"2.117000016612674"} \qquad \exp(a) = 2.117000016612670$

bigQ Math

Doing big integer algebra Digits := 40

Smath expression $N := \dfrac{\sqrt{3} - 1}{2 \cdot \sqrt{2}}$

Authomatica conversion to bigQ algebra $R := bigQN2E\left(\text{num2str}(N)\right) = bigQDiv\left(bigQAdd\left(\left[\begin{matrix}\text{"-1"} \\ \text{"1"}\end{matrix}\right], bigQPow\left(\left[\begin{matrix}\text{"3"} \\ \text{"1"}\end{matrix}\right], bigQDiv\right.\right.\right.$

Apply eval $R := \text{eval}(R)$

Big integer $R = \left[\begin{matrix}\text{"157056521839177752903510227009491133369240092117876996755798360116} \\ \text{"606819802526379479300287016906944139497147450559048314194109621079}\end{matrix}\right.$

Decimal expression

$N = 0.258819045102521$ $D := bigQ2D(R) = \text{"0.2588190451025207623488988376240483283 49"}$

Remember that the actual big rational is R with the couple of integers, and D is only for know what it represent. Some other examples

- $N := \sin\left(\dfrac{\pi}{12}\right)$ $R := bigQN2E\left(\text{num2str}(N)\right)$ $R := \text{eval}(R) = \left[\begin{matrix}\text{"543984589299785499582818} \\ \text{"210179505563165905421803 1}\end{matrix}\right.$

  $N = 0.258819045102521$ $D := bigQ2D(R) = \text{"0.2588190451025207623488988376240483283 49"}$

- $N := \dfrac{1 + \sqrt{5}}{2}$ $R := bigQN2E\left(\text{num2str}(N)\right)$ $R := \text{eval}(R) = \left[\begin{matrix}\text{"652393943937479691747801} \\ \text{"403201631407955854118492}\end{matrix}\right.$

  $N = 1.61803398874989$ $D := bigQ2D(R) = \text{"1.6180339887498948482045868343656381177 20"}$

- $N := 2 \cdot \cos\left(\dfrac{\pi}{5}\right)$ $R := bigQN2E\left(\text{num2str}(N)\right)$ $R := \text{eval}(R) = \left[\begin{matrix}\text{"523021871795318557495976} \\ \text{"323245293629096854977274}\end{matrix}\right.$

  $N = 1.61803398874989$ $D := bigQ2D(R) = \text{"1.6180339887498948482045868343656381177 20"}$

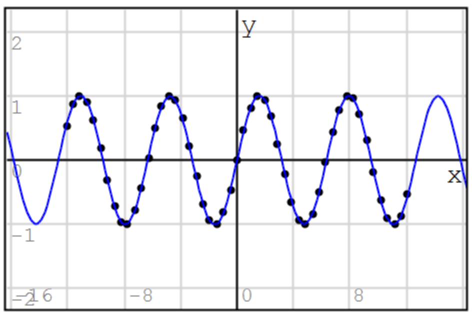
- $N := \ln\left(1 + \mathbf{e}^2\right)$ $R := bigQN2E\left(\text{num2str}(N)\right)$ $R := \text{eval}(R) = \left[\begin{matrix}\text{"216353086268759246624969} \\ \text{"101720925741472142615593}\end{matrix}\right.$

  $N = 2.12692801104297$ $D := bigQ2D(R) = \text{"2.1269280110429724964437268063583044314 34"}$

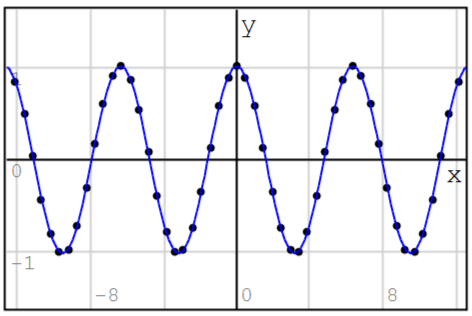
Functions Plots

Digits := 15 $Plot(fs, gs, a, b) := \left\| \begin{cases} gs \\ \text{augment}\left(X := a + \dfrac{b - a}{50} \cdot [0..50], \overrightarrow{bigQ2N\left(feval(fs, X)\right)}, \text{"."}\right) \end{cases}\right.$

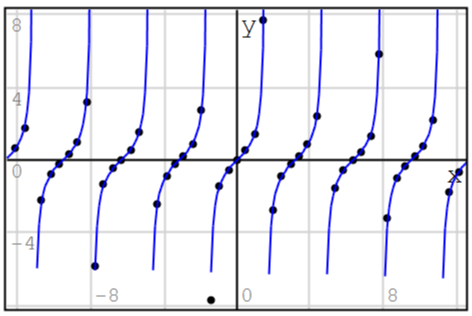
Notice that trigonometric functions Sin, Cos and Tan are well defined in the interval -2π..2π. You can extend outside of that interval using bigQMod.



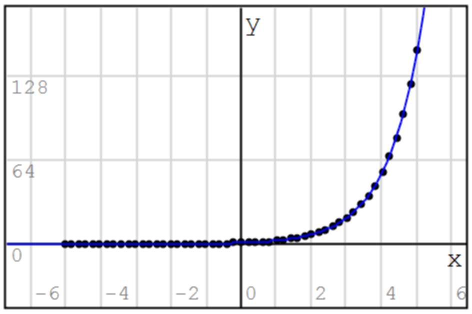
$Plot(\text{"bigQSin"}, \sin(x), -12, 12)$



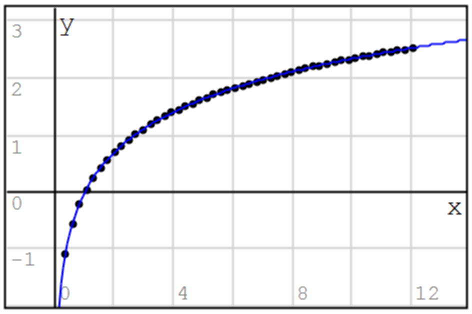
$Plot(\text{"bigQCos"}, \cos(x), -12, 12)$



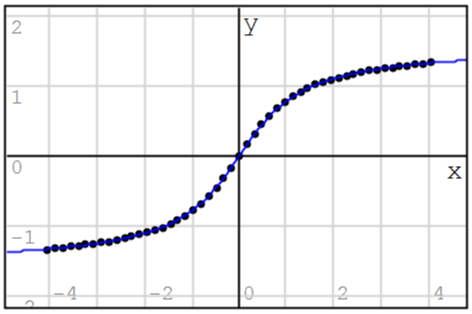
$Plot(\text{"bigQTan"}, \tan(x), -12, 12)$



Plot ("bigQExp", exp (x), − 5, 5)



Plot ("bigQLog", ln (x), 0.1, 12)



Plot ("bigQATan", atan (x), − 4, 4)

Gallery

Digits := 400

$R_1 := bigQExp(1)$

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2.71828182845904523536028747135266249775
7247093699959574966967627724076630353547
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2.71828182845904523536028747135266249775
7247093699959574966967627724076630353547
5945713821785251664274274663919320030599
2181741359662904357290033429526059563073
8132328627943490763233829880753195251019
0115738341879307021540891499348841675092
4476146066808226480016847741185374234544
2437107539077744992069551702761838606261
3313845830007520449338265602976067371132
0070932870912744374704723069697720931014

Show (10, 0.1 Digits, bigQ2D (R_1))

$N_2 := 16 \cdot \text{atan}\left(\frac{1}{5}\right) - 4 \cdot \text{atan}\left(\frac{1}{239}\right)$

$R_2 := bigQN2E\left(\text{num2str}\left(N_2\right)\right)$

3.14159265358979323846264338327950288419
7169399375105820974944592307816406286208
9986280348253421170679821480865132823066
4709384460955058223172535940812848111745
0284102701938521105559644622948954930381
9644288109756659334461284756482337867831
6527120190914564856692346034861045432664
8213393607260249141273724587006606315588
1748815209209628292540917153643678925903
6001133053054882046652138414695194151160

Show (10, 0.1 Digits, bigQ2D (R_2))

Continued fractions

$a2(n, x) := [\,"1"\;\;"1"\,]$

$b2(n, x) := \begin{cases} [\,"1"\;\;"1"\,] & \text{if} \quad n = 0 \\ [\,"2"\;\;"1"\,] & \text{otherwise} \end{cases}$

$R_1 := bigQCF("a2", "b2", 0)$

1.41421356237309504880168872420969807856
9671875376948073176679737990732478462107
0388503875343276415727350138462309122970
2492483605585073721264412149709993583141
3222665927505592755799950501152782060571
4701095599716059702745345968620147285174
1864088919860955232923048430871432145083
9762603627995251407989687253396546331808
8296406206152583523950547457502877599617
2983557522033753185701135437460340849884

Show (10, 0.1 Digits, bigQ2D (R_1))

$a\pi(n, x) := \begin{cases} bigQ(4) & \text{if} \quad n = 1 \\ bigQ\left((n-1)^2\right) & \text{otherwise} \end{cases}$

$b\pi(n, x) := \begin{cases} bigQ(0) & \text{if} \quad n = 0 \\ bigQ(2 \cdot n - 1) & \text{otherwise} \end{cases}$

$R_2 := bigQCF(a\pi, b\pi, x)$

3.14159265358979323846264338327950288419
7169399375105820974944592307816406286208
9986280348253421170679821480865132823066
4709384460955058223172535940812848111745
0284102701938521105559644622948954930381
9644288109756659334461284756482337867831
6527120190914564856692346034861045432664
8213393607260249141273724587006606315588
1748815209209628292540917153643678925903
6001133053054882046652138414695194151160

Show (10, 0.1 Digits, bigQ2D (R_2))

Alvaro