A long uniform rod of 50-mm diameter with a thermal conductivity of 15 W/m-K is heated internally by volumetric energy generation of 20 kW/m3. The rod is positioned coaxially within a larger circular tube of 60-mm diameter whose surface is maintained at 500 degC. The annular region between the rod and the tube is evacuated, and their surfaces are diffuse and gray with an emissivity of 0.2.
a) Determine the center and surface temperatures of the rod.
b) Determine the center and surface temperatures of the rod if atmospheric air occupies the annular space.
c) For tube diameters of 60, 100, and 1000 mm and for both the evacuated and atmospheric conditions, compute and plot the center and surface temperatures as a function of equivalent surface emissivities in the range from 0.1 to 1.0.

$D_1 := 50\ mm$ $\qquad k := 15\ \dfrac{W}{m \cdot K}$ $\qquad q' := 20\ \dfrac{kW}{m^3}$

$T_2 := 773\ K$ $\qquad T_2 = 499.85\ °C$ $\qquad F_{12} := 1$ $\qquad \sigma := 5.67 \cdot 10^{-8}\ \dfrac{W}{m^2 \cdot K^4}$ Stefan Boltzmann constant:

$A_1 := \pi \cdot D_1 = 0.157\ m$ $\qquad A_2(D_2) := \pi \cdot D_2$

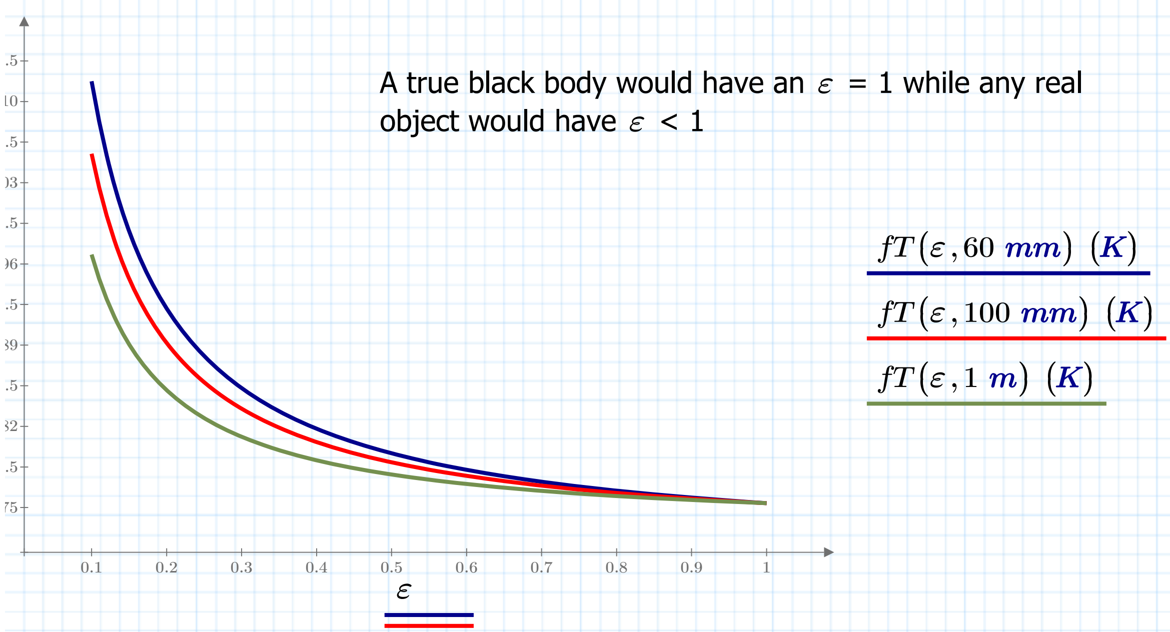
$T := 300\ K$

$$\frac{\sigma \cdot \left(T^4 - {T_2}^4\right)}{\dfrac{1-\varepsilon}{\varepsilon \cdot A_1} + \dfrac{1}{F_{12} \cdot A_1} + \dfrac{1-\varepsilon}{\varepsilon \cdot A_2(D_2)}} = q' \cdot \left(\frac{\pi \cdot {D_1}^2}{4}\right)$$

$fT(\varepsilon, D_2) := \mathbf{find}(T)$

$\varepsilon := 0.1, 0.11 .. 1$ Range variable used only for plotting!

A true black body would have an ε = 1 while any real object would have ε < 1



If you need the vectors you created for further calculations ...

$i := 0..9 \qquad \varepsilon_i := \frac{i+1}{10} \qquad \varepsilon^{T} = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 \end{bmatrix}$

$T_{60} := \overrightarrow{fT(\varepsilon, 60\ mm)}^{T} = \begin{bmatrix} 538.6 & 519.01 & 512.15 & 508.65 & 506.52 & 505.1 & 504.08 & 503.31 & 502.71 & 502.23 \end{bmatrix}\ °C$

OR

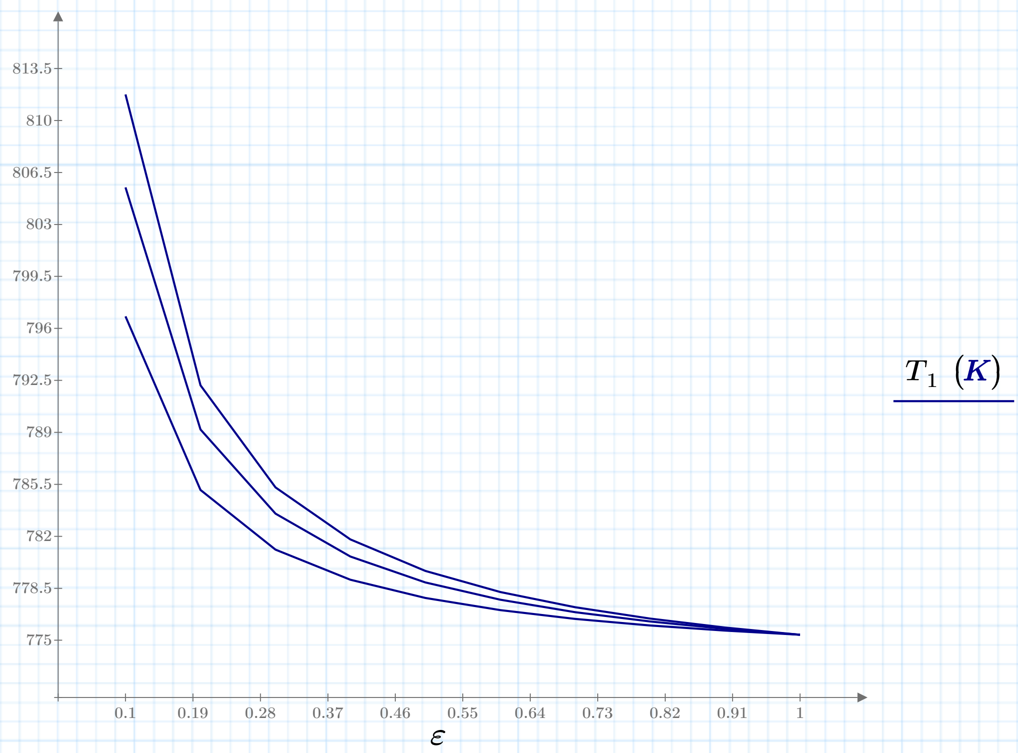
$D_2 := \text{stack}(60\ mm, 100\ mm, 1\ m)$

$d := 0..\text{last}(D_2)$

$T_1^{\langle d \rangle} := \overrightarrow{fT\left(\varepsilon, D_{2_d}\right)}$

$$T_1 = \begin{bmatrix} 538.601 & 532.347 & 523.665 \\ 519.013 & 516.04 & 511.972 \\ 512.149 & 510.372 & 507.955 \\ 508.648 & 507.492 & 505.923 \\ 506.525 & 505.748 & 504.697 \\ 505.1 & 504.579 & 503.875 \\ 504.077 & 503.741 & 503.287 \\ 503.307 & 503.111 & 502.845 \\ 502.707 & 502.619 & 502.501 \\ 502.226 & 502.226 & 502.226 \end{bmatrix}\ °C$$

You may even create a waterfall plot of that matrix, but with the stepwidth you have defined in eps (0.1) and all curves are one plot, so they share the same color.



You can use T in your various other calculations, like

$$T_{mid} := \frac{q' \cdot \left(\frac{D_1}{2}\right)^2}{4 \cdot k} + T_1 \qquad T_{mid} = \begin{bmatrix} 538.809 & 532.555 & 523.874 \\ 519.221 & 516.248 & 512.18 \\ 512.357 & 510.581 & 508.163 \\ 508.856 & 507.7 & 506.132 \\ 506.733 & 505.957 & 504.905 \\ 505.308 & 504.788 & 504.084 \\ 504.285 & 503.95 & 503.496 \\ 503.515 & 503.319 & 503.054 \\ 502.915 & 502.828 & 502.71 \\ 502.434 & 502.434 & 502.434 \end{bmatrix} {}^\circ C$$

every column in that matrix represents one of the diameters in D2: 60mm, 100mm & 1000mm

$$\alpha := 115.6 \cdot 10^{-6} \ \frac{m^2}{s} \qquad v := 81.5 \cdot 10^{-6} \ \frac{m^2}{s} \qquad k_{cond} := 0.0563 \ \frac{W}{m \cdot K}$$

$$L_c := 2 \cdot \frac{\ln\left(\frac{D_2}{D_1}\right)^{\frac{4}{3}}}{\left(\left(\frac{D_1}{2}\right)^{\frac{-3}{5}} + \left(\frac{D_2}{2}\right)^{\frac{-3}{5}}\right)^{\frac{5}{3}}} = \begin{bmatrix} 0.00178 \\ 0.01318 \\ 0.16723 \end{bmatrix} m$$

$$\beta := \frac{1}{\frac{T_1 + T_2}{2}} = \begin{bmatrix} 0.00126 & 0.00127 & 0.00127 \\ 0.00128 & 0.00128 & 0.00128 \\ 0.00128 & 0.00128 & 0.00129 \\ 0.00129 & 0.00129 & 0.00129 \\ 0.00129 & 0.00129 & 0.00129 \\ 0.00129 & 0.00129 & 0.00129 \\ 0.00129 & 0.00129 & 0.00129 \\ 0.00129 & 0.00129 & 0.00129 \\ 0.00129 & 0.00129 & 0.00129 \\ 0.00129 & 0.00129 & 0.00129 \end{bmatrix} \frac{1}{K}$$

$$Ra_c^{\langle d \rangle} := \frac{\overrightarrow{g \cdot \beta^{\langle d \rangle} \cdot \left(T_1^{\langle d \rangle} - T_2\right) \cdot {L_c}_d^{\,3}}}{v \cdot \alpha} \qquad Ra_c = \begin{bmatrix} 0.287 & 98.18 & 1.477 \cdot 10^5 \\ 0.144 & 49.423 & 7.574 \cdot 10^4 \\ 0.093 & 32.239 & 5.078 \cdot 10^4 \\ 0.066 & 23.458 & 3.81 \cdot 10^4 \\ 0.05 & 18.126 & 3.043 \cdot 10^4 \\ 0.04 & 14.544 & 2.528 \cdot 10^4 \\ 0.032 & 11.973 & 2.16 \cdot 10^4 \\ 0.026 & 10.037 & 1.883 \cdot 10^4 \\ 0.022 & 8.527 & 1.667 \cdot 10^4 \\ 0.018 & 7.317 & 1.494 \cdot 10^4 \end{bmatrix}$$

$$q'_{rad}\left(T,\varepsilon,D_2\right):=\frac{\sigma\cdot\left(T^4-{T_2}^4\right)}{\dfrac{1-\varepsilon}{\varepsilon\cdot A_1}+\dfrac{1}{F_{12}\cdot A_1}+\dfrac{1-\varepsilon}{\varepsilon\cdot A_2\left(D_2\right)}}$$

$$q'_{cond}\left(T,\varepsilon,D_2\right):=\frac{2\cdot\pi\cdot k_{cond}\cdot\left(T-T_2\right)}{\ln\left(\dfrac{D_2}{D_1}\right)}$$

Guess Values

$$T:=300\ \boldsymbol{K}$$

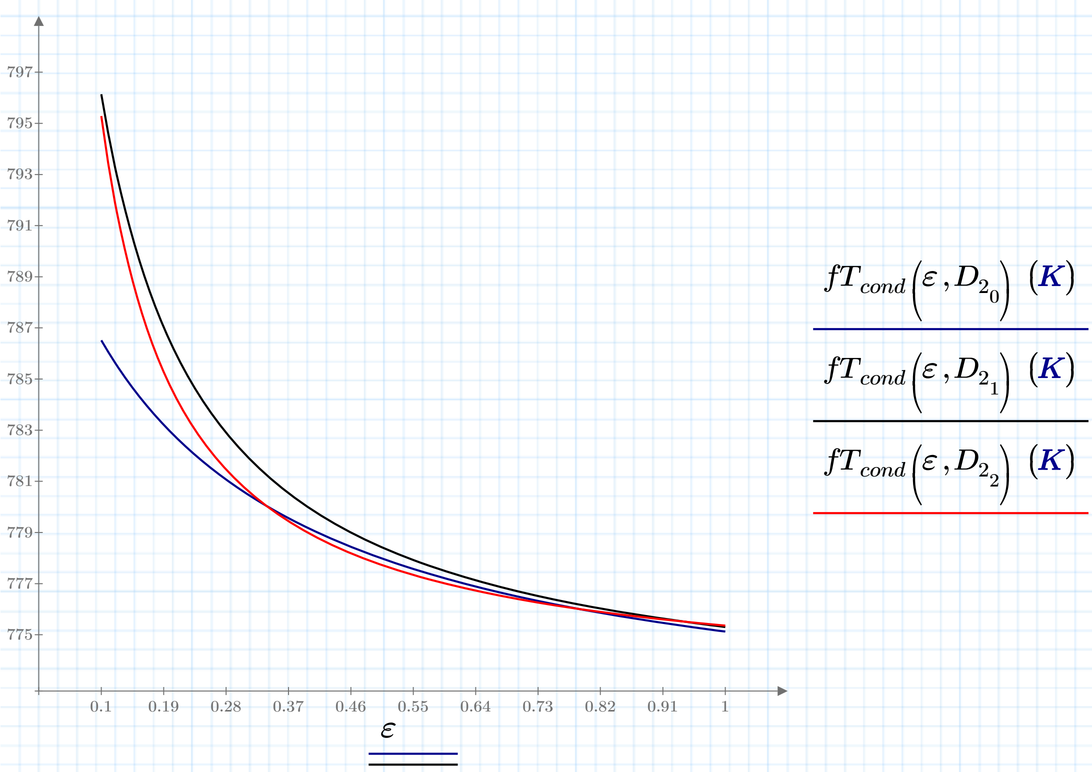
Constraints

$$\frac{4}{\pi\cdot{D_1}^2}\cdot\left(q'_{rad}\left(T,\varepsilon,D_2\right)+q'_{cond}\left(T,\varepsilon,D_2\right)\right)=q'$$

Solver

$$fT_{cond}\left(\varepsilon,D_2\right):=\mathbf{Find}\left(\mathrm{T}\right)$$

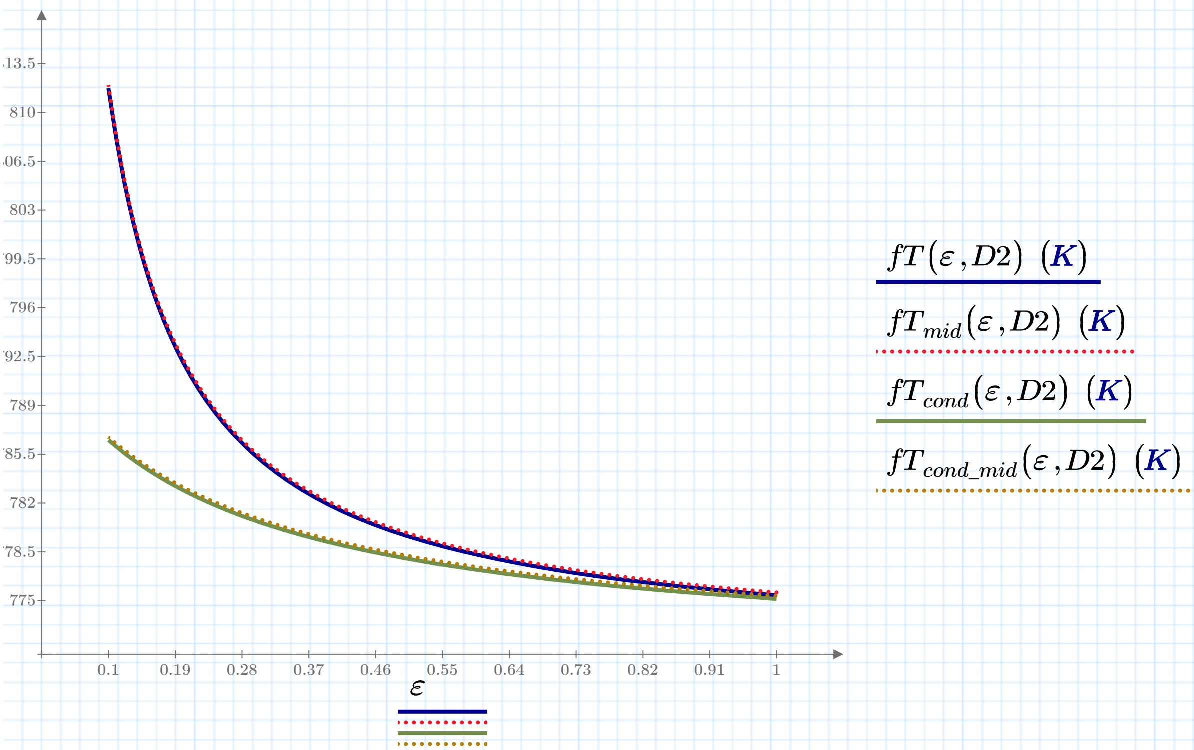
$$\varepsilon:=0.1,0.11..1$$



$$fT_{cond\_mid}\left(\varepsilon, D_2\right) := \frac{q' \cdot \left(\frac{D_1}{2}\right)^2}{4 \cdot k} + fT_{cond}\left(\varepsilon, D_2\right)$$

$$fT_{mid}\left(\varepsilon, D_2\right) := \frac{q' \cdot \left(\frac{D_1}{2}\right)^2}{4 \cdot k} + fT\left(\varepsilon, D_2\right)$$

$D2 := 60\ mm$



$fT(0.2, D2) = 519.013\ °C$

$fT_{mid}(0.2, D2) = 519.221\ °C$

$fT_{cond}(0.2, D2) = 509.784\ °C$

$fT_{cond\_mid}(0.2, D2) = 509.992\ °C$